

Errata

Correction to equation 2.6 on p. 38

$$n_{MP} \geq 2 \cdot t_{rt} / t_{fix} - 1 \quad (2.6)$$

Correction to the example on p. 70/71 and 73/74

$$105.5959 - 1.50635 \cdot 70 + 0.00650599 \cdot 70^2 \text{ kN.}$$

The weight of the automotive train is 116 t. The resistance of the automotive train is assumed to be $1.39 \cdot m[t] \cdot g + 0.00905 \cdot m[t] \cdot v[\text{km/h}] \cdot g + 0.0296 v[\text{km/h}]^2$ measured in Newton.

We would like to determine the time to speed up the automotive train from 40 km/h to 70 km/h. The gradient is assumed to be zero.

First we have to transform the speeds into m/s. Since $1 \text{ km/h} = 1/3.6 \text{ m/s}$ and therefore $1 \text{ m/s} = 3.6 \text{ km/h}$, the interval borders have to be divided by 3.6 to get them as speeds in m/s. $c_{1,k}$ has to be multiplied by 3.6, and $c_{2,k}$ has to be multiplied by $3.6^2=12.96$. Finally, the coefficients have to be multiplied by 1000 to get the tractive effort in Newton. We get the following table.

Speed interval in m/s	$c_{0,k}$	$c_{1,k}$	$c_{2,k}$
0-3.166	128464.4	-5310.6	-58.86
3.166-8.333	132274.1	-7634.6	13.86
8.333-16.667	132470.2	-7778.7	155.58
16.667-22.167	105595.9	-5422.9	84.32
22.167-27.500	20089.1	1087.0	-260.38
27.500-32.500	50550.4	-560.1	-6.01
32.500-38.889	-4107	1741.4	-27.25
38.889-44.444	35080.9	-90.0	-6.48

The intervals that are affected are 30-60 km/h and 60-79.8 km/h. When we transform the speed measure to m/s, we consider the interval 40-60 km/h that is identical to the interval 11,1 to 16,667 m/s and the interval 60-70 km/h that is identical to the interval 16.667 to 19.4 m/s.

In the interval 11,1 to 16.667 m/s, the tractive effort in Newton is

$$132470.2 - 7778.7 v + 155.58v^2$$

and in the interval 16.667 to 19.4 m/s, the tractive effort is

$$105595.9 - 5422.9 v + 84.32 v^2.$$

Taking into account that the gravitation constant g is 9.81 m/s^2 , the resistance is

$$1581.8 + 37.075 v + 0.3836 v^2.$$

$F_{Tr} - F_R$ is therefore

$$130888.4 - 7815.775 v + 155.1964 v^2$$

in the interval 11.1 to 16.667 m/s and

$$104014.1 - 5459.975 v + 83.9364 v^2$$

in the interval 16.667 to 19.4 m/s.

The mass factor is given by $f_p = 1.09$. Then the following differential equations apply to express the relation between time and speed during the acceleration phase:

$$130888.4 - 7815.775 v + 155.1964 v^2 = 116000 \cdot 1.09 \cdot dv/dt$$

for the interval 11.1 to 16.667 m/s and

$$104014.1 - 5459.975 v + 83.9364 v^2 = 116000 \cdot 1.09 \cdot dv/dt$$

for the interval 16.6 to 19.4 m/s.

Example continued

We are given the following differential equations to describe the tractive force.

$$130888.4 - 7815.775 v + 155.1964 v^2 = 116000 \cdot 1.09 \cdot dv/dt$$

for the interval 11.1 to 16.667 m/s and

$$104014.1 - 5459.975 v + 83.9364 v^2 = 116000 \cdot 1.09 \cdot dv/dt$$

for the interval 16.667 to 19.4 m/s.

We would like to determine the time and the length of the path to accelerate from 11.1 m/s to 19.4 m/s.

We restrict to the acceleration from 11.1 m/s to 16.67 m/s (that is around 16.7). The difference is 5.6, and an appropriate value for Δv is 1.4. Equation (4.26) can be written as follows.

$$t_{i+1} = 116000 \cdot 1.09 \cdot 1.4 / (130888.4 - 7815.775 v_i + 155.1964 v_i^2) + t_i$$

Equation (4.28) is now as follows.

$$s_{i+1} = 116000 \cdot 1.09 \cdot 1.4 \cdot v_i / (130888.4 - 7815.775 v_i + 155.1964 v_i^2) + s_i.$$

The following table shows the time to accelerate from 11.1 m/s to 16.7 m/s and all intermediate values v_i , t_i , and s_i .

i	v_i	t_i	s_i
0	11.1	0	0
1	12.5	2.8	31.02
2	13.9	5.9	69.5
3	15.3	9.6	126.4
4	16.7	13.7	194.1

In 13.7 seconds the train passes a path of length 194.1 m if it speeds up from 11,1 m/s = 40 km/h to 16.7 m/s = 60 km/h.

We compute the time and the path length to accelerate from 16.6 m/s to 19.4 m/s = 70 km/h in the same way. This may be left as an exercise.